Artigo

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Sérgio Shiguemi Furuie
(Divisão de Informática – INCOR / HCFM – USP)

Nelson Delfino D' Ávila Mascarenhas
(DPI–INPE)

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TOMOGRAPHIC RECONSTRUCTION OF IMAGES WITH POISSON NOISE: PROJECTION ESTIMATION

Sérgio Shiguemi Furuie, Nelson Delfino D'Ávila Mascarenhas*
Divisão de Informatica - Instituto do Coração/HCFMUSP
Av.Dr.Eneas de Carvalho Aguiar, 44
CEP 05403 Sao Paulo SP
*Departamento de Processamento de Imagens - INPE

ABSTRACT-This work presents an alternative approach to reconstructing images with low signal-to-noise ratio. It consists of estimating projections assuming that the noise is Poisson and reconstructing the image applying transform methods. The proposed method yields results similar to ML-EM, but using much less processing time (1 to 2 orders).

1.INTRODUCTION

Computerized Tomography (CT) has been applied to many fields, involving from molecular dimensions (electronic microscopy) to cosmic dimensions (radio astronomy) (Herman, 1980). Its ability to visualize internal structures, such as transverse sections of the human body, made CT in an invaluable instrument for medicine.

The speed and simplicity of transform methods for reconstruction have led to their widespread use in tomography. However, for projections with poor signal-to-noise ratio, direct application of transform methods yields unacceptable results. On the other hand statistical methods such as Maximum Likelihood implemented by the Expectation-Maximization technique (ML-EM) produces better results (Chornoboy, 1990) in Emission Computerized Tomography (ECT), but it is rather time consuming.

In this paper, we present a fast alternative technique for reconstructing images with Poisson noise and low signal-to-noise ratio. The proposed method is based on estimating projections considering a model for noise and projection formation.

2.MODEL

We will assume the model depicted in figure 1, where $\underline{f}_N^T =[f_1,f_2...f_N]$ represents the object image that is subjected to independent Poisson process resulting in a random field denoted by $\vec{X}_N^T=[X_1,X_2,\ldots X_N]$. The noisy projections ($\vec{Y}_M$) are obtained by applying the projection operator H_{MxN} on $\vec{X}$. Therefore the following expressions are valid (Lo, 1979) (capital letters for X,Y,Z and G denote random variables):

$$\text{Prob}[\vec{X}=\vec{x}\ |\vec{f}\ ]=\prod_{j=1}^{N}\text{Prob}[X_j=x_j|f_j]=\prod_{j=1}^{N}\frac{e^{-f_j}.(f_j)^{x_j}}{x_j!} \qquad (1)$$

$$E[X_j|f_j]=\text{var}[X_j|f_j]=f_j \qquad (2)$$

$$\vec{Y}_M=H_{MxN}.\vec{X}_N \qquad (3)$$

where H :projection matrix that can encompass point spread function, geometric characteristic of the acquisition device, attenuation and scattering correction.

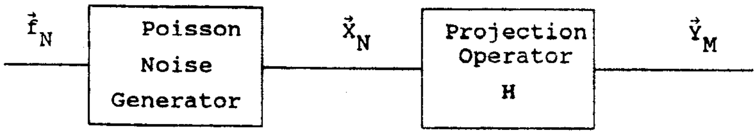


Figure 1. Model for formation of noisy projections ($\vec{Y}_M$) where the object image ($\vec{f}_N$) corresponds to the average rate of a Poisson process ($\vec{X}_N$)

It can be shown that (Y_i , i=1,M) are independent random variables with Poisson distribution (Vardi, 1985). Therefore,

$$E[Y_i|\vec{f}_N]=\text{var}[Y_i|\vec{f}_N]=\sum_{j=1}^{N}h_{ij}.f_j \qquad i=1,M \qquad (4)$$

$$E[\vec{Y}_M|\vec{f}_N]=H.\vec{f}_N \qquad (5)$$

Making $\vec{g}_M= H.\vec{f}_N$, and considering the equations above, we can first estimate $\vec{g}$ and then reconstruct $\vec{f}$ as shown in figure 2.

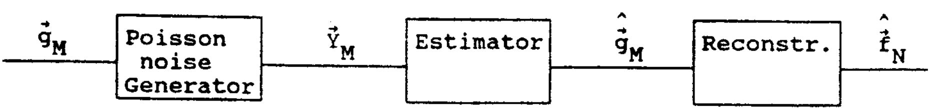


Figure 2. Model for estimation and reconstruction

3. METHOD

The proposed technique consists basically of using an optimized estimator to obtain $\hat{\vec{g}}_M$ given $\vec{y}_M$, considering the Poisson nature of the noise.

MAP estimator (Lo, 1979)

Assuming that $\{G_i, i=1,M\}$ are uncorrelated random variables with Gaussian distribution, and the distribution of $\vec{Y}$ given $\vec{g}$ (figure 2) is a Poisson process, the MAP (Maximum a Posteriori) estimator is derived by

$$\max_{\vec{g}} Prob[\vec{G}=\vec{g} \mid \vec{y}] = \max \frac{Prob[\vec{Y} \mid \vec{g}].Prob[\vec{G}]}{Prob[\vec{Y}]} \qquad (6)$$

yielding

$$\hat{g}_i = \frac{E(G_i)-var(G_i)+\sqrt{(E(G_i)-var(G_i))^2+4.var(G_i).y_i}}{2} \qquad i=1,M \qquad (7)$$

Wiener estimator (Kuan, 1985)

Assuming the same hypothesis of MAP estimator, the Wiener estimator can be obtained by

$$\min_B E[\|\vec{G} - \hat{\vec{g}}\|^2] \quad \text{where } \hat{\vec{g}}=E(\vec{G})+B.(\vec{y}-E(\vec{Y})) \qquad (8)$$

resulting in a punctual estimator (adaptive filter)

$$\hat{g}_i = E(G_i) + \frac{var(G_i)}{var(G_i)+E(G_i)}.(y_i - E(G_i)) \qquad i=1,M \qquad (9)$$

Anscombe Transformation

Alternatively, the Anscombe transformation (Anscombe, 1948) can be applied to $\vec{Y}_M$ in order to transform Poisson noise in approximately Gaussian noise with variance equal to 1. Henceforth we can use MAP estimator assuming that the noise is Gaussian.

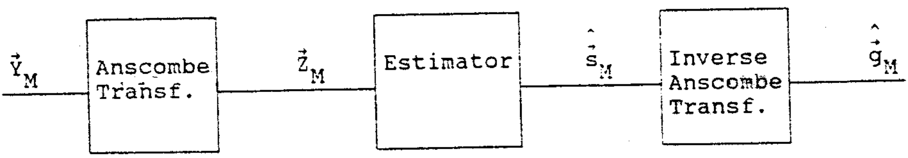


Figure 3. Diagram used for estimation with Anscombe transformation

Using a similar approach, it can be demonstrated that MAP and Wiener estimators coincide and are given by

$$\hat{s}_i=\beta_i z_i+(1-\beta_i)E(S_i) \qquad i=1,M \qquad (10)$$

$$\beta_i=\frac{var(S_i)}{var(Z_i)+var(S_i)} \qquad (11)$$

where $var(Z_i)=1$ (12)

In equation (10), if we estimate $E(\vec{S})$ using smoothed noisy data, the estimated value $\hat{s}$ is a linear combination of the available and the smoothed data. A heuristic filter (Maeda, 1987) can be formulated using the local median instead of z_i.

<u>Heuristic</u> <u>estimator</u>

$$s_i=\beta_i.\tilde{z}_i+(1-\beta_i)\bar{z}_i \qquad (14)$$

where $\tilde{z}$:local median

$\bar{z}$: local average

$$\beta=\frac{\text{local variance}}{\text{maximum variance}}$$

This filter has interesting properties such as preserving edges due to median filter and use of local statistics.

<u>Minimum</u> <u>cost</u> <u>estimator</u>

If we define

$$cost(\vec{s}) = ||\vec{z}-\vec{s}||^2+\beta.||L\vec{s}||^2 \qquad (15)$$

where $(L\vec{s})_i = s_{i-1}-2\,s_i+s_{i+1}$ (16)

This function involves the distance between the input data and the estimated data as well as a measure of roughness. Minimizing this function in $\vec{s}$, we have, in frequency domain (where T is sample distance):

$$S(w)=\frac{Z(w)}{1+\beta.(2\cos 2wT-8\cos wT+6)} \qquad (17)$$

S(w) is a low-pass band filter, with parameter β for roughness effect. The response of this filter in the frequency domain is illustrated in figure 4 for β=0.01, 0.1,1,10 and 100.

In order to assess these estimators, simulated data with 512 samples was generated. Table 1 summarizes the results where square root of normalized mean square error (NMSE) was used as a measure of estimation quality (figure 2):

$$\mathrm{NMSE} = \sqrt{\sum(\hat{g}_i - g_i)^2 / \sum g_i^2} \qquad (18)$$

Reconstruction

It was assumed that the projections are Radon transform of the object image. Thus the image can be reconstructed using FBP (Filtering-Backprojection) applied to estimated projections. The results are compared with simple FBP and ML-EM.

4. RESULTS

For quantitative evaluation we used a circle-like image (radius R=8 pixels), as shown in figure 5, divided into 32x32 pixels, generating 64 projection views, 32 samples on each projection view. Total count was 10000, resulting in 49.7 counts in average for regions inside the circle, and a maximum of 26.0 for noisy projections.

The results are shown in figure 6 and table 2, where NMSE was calculated in 3 regions of the circle: a)central (inner circle with radius equal to 70% of R); b) edge (ring delimited by circumferences of radius 0.7R and 1.3R); and c)global. Estimation and reconstruction time was also measured. This table shows that the proposed technique produces results comparable to ML-EM, but spending only a fraction of ML-EM time.

5. CONCLUSIONS

It has been shown that the reconstruction of images with independent Poisson noise can be done more efficiently in two steps: estimation and reconstruction. The proposed estimator is a heuristic filter (Maeda) applied to transformed projections by the Anscombe transform. If the projections can be approximated by a Radon transform of the original image, a conventional Filtering-Backprojection algorithm can be used for reconstruction. We have shown that the proposed approach leads to results comparable to ML-EM, but with a much lower computating time.

Table 1. Performance of discussed estimators in relation to error and computational time

	NMSE	time* (s)
MAP	0.1123	1.10
Wiener	0.1121	1.04
Anscombe		
MAP	0.1113	1.38
Heuristic	0.0958	1.81
Cost(β=1)	0.1005	6.86

*Based on IBM PC-AT at 6MHz, with 80287 co-processor

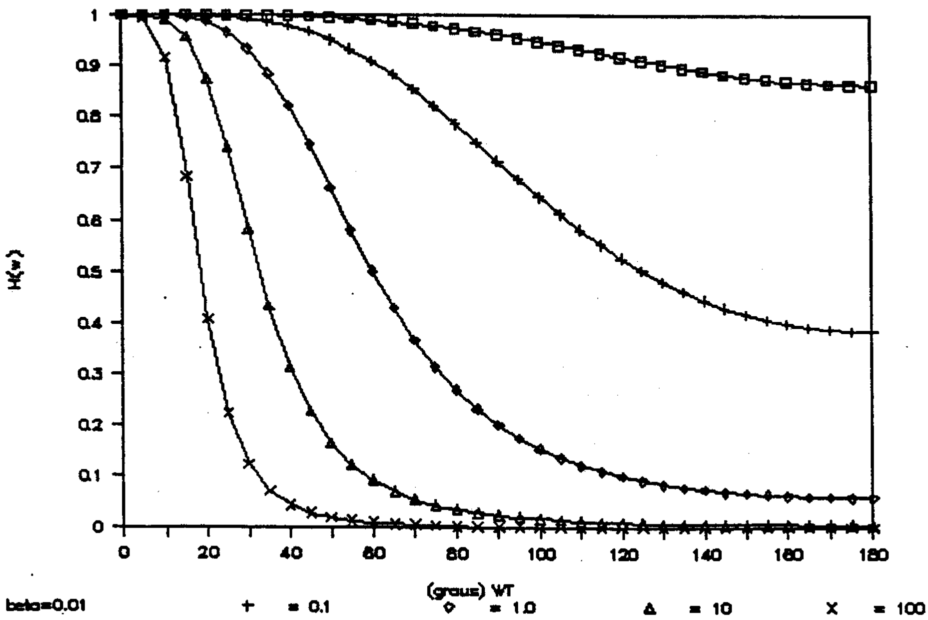
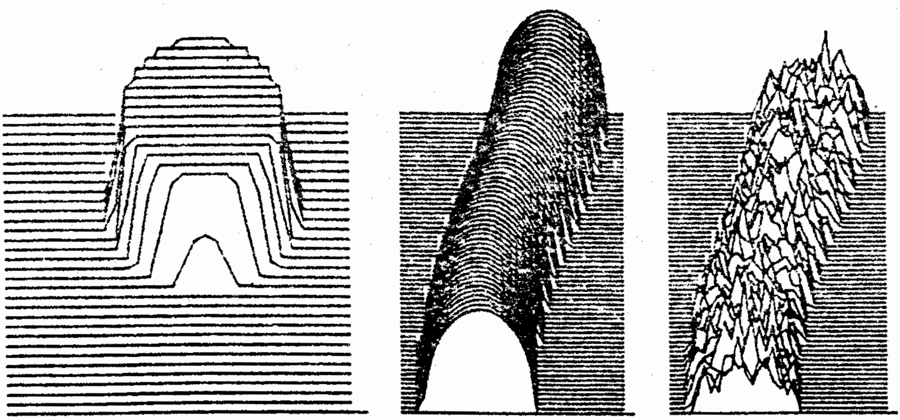


Figure 4. Frequency response of a filter based on minimum cost for β=0.01, 0.1, 1.0, 10 and 100

Table 2. Reconstruction performance of simple Filtering-Backprojection (FBP), proposed technique and ML-EM method with 5 iterations using noisy projections (figure 5c).

	NMSE			Time* (s)
	Central	Edge	Global	
FBP	0.1383	0.2692	0.2660	73.0
Proposed	0.0493	0.2474	0.1875	82.12
ML-EM	0.0954	0.2504	0.1905	2370.0

*Based on IBM-PCAT, at 6MHz, with 80287 co-processor



a) b) c)

Figure 5, a) Simulated object image (32x32 pixels); b) sinogram with 64 projection views and 32 samples each; c) noisy sinogram (Poisson noise).

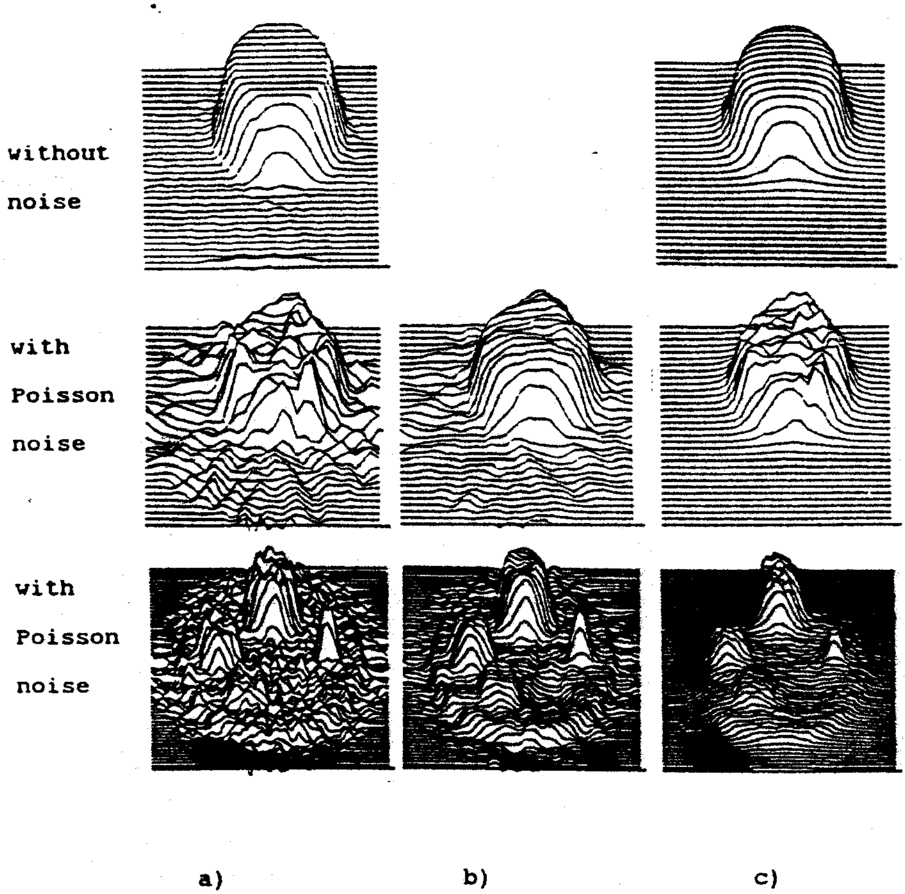


Figure 6.Reconstruction results based on: a) simple Filtering-Backprojection; b)proposed technique; c)ML-EM with 5 iterations, for circle with and without noise, and Tanaka's figure [9]

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